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EXPERIMENTAL VIBRATION CHARACTERISTICS OF A BEAM WITH NONLINEAR SUPPORT USING RECEPTANCE COUPLING ANALYSIS

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Resumo. *This paper applies the Multi-Harmonic Nonlinear Receptance Coupling Approach (MUHANORCA) (Ferreira 1998) to evaluate the frequency response characteristics of a beam which is clamped at one end and supported at the other end by a nonlinear cubic stiffness joint. In order to apply the substructure coupling technique, the problem was characterised by coupling a clamped linear beam with a nonlinear cubic stiffness joint. The experimental results were obtained by a sinusoidal excitation with a special force control algorithm where the level of the fundamental force is kept constant and the level of the harmonics is kept zero for all the frequencies measured.*

Keywords: *Experimental, Substructuring, Receptance, Coupling, Nonlinear*

1. INTRODUCTION

Although the various phenomena of nonlinear oscillations have long been recognised by many scientists, the practical solution of nonlinear problems has only been stimulated by the growing development in computers. After the computers, a large volume of investigation was carried out using the classical time domain techniques. However, as a means of obtaining the steady-state response solution, these methods are very time consuming. Therefore, current efforts are directed towards the development of approximate frequency domain methods to seek approximate solutions for the nonlinear vibration problem. Many of these approximate frequency domain methods determine the steady-state first order harmonic response of structures using the describing function technique (Cameron & Griffin 1989, Comert & Ozguven 1995, Ferreira & Ewins 1996, Lin 1988, Murakami & Sato

1990, Tanrikulu *et al.* 1993, Watanabe & Sato 1988). Some of these methods improved the dynamic response analyse by using a higher order of harmonics in the response (Ferreira & Ewins 1997, Kuran & Ozguven 1996, Pierre *et al.* 1985, Ren & Beards 1993, Ren 1992).

The object of the research reported here has been to provide, through detailed experimental studies, insight into accuracy of theoretical predictions by the application of the *MUHANORCA* method previously developed by Ferreira & Ewins (1997), which is an approximated frequency domain method based in harmonics of the response. During the modelling, due to different conditions which apply to the analytical and experimental configuration, special concerns have been taken to simulate experimentally the theoretical problem as precisely as possible. One concern was to reproduce experimentally the special excitation force used in the theoretical analysis characterised by a pure constant harmonic force. This special excitation was achieved experimentally by using an exciter together with a nonlinear force control algorithm that assures the pure harmonic excitation. Another concern was with the limitations of reproducing analytically the experimental boundary conditions. Therefore the solution was to updated the analytical model in order to represent the experimental boundaries conditions obtained.

2. Experimental Test Rig

2.1. Test Rig Model

The Test Rig was made of a continuous system having a local nonlinear cubic stiffness, as shown in Figure 1. It consists of a uniform beam, A, of 420 mm length with a cross

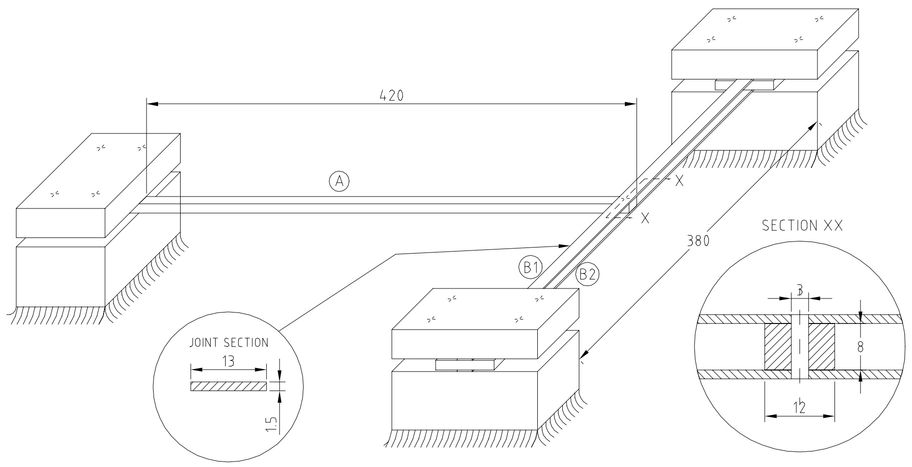


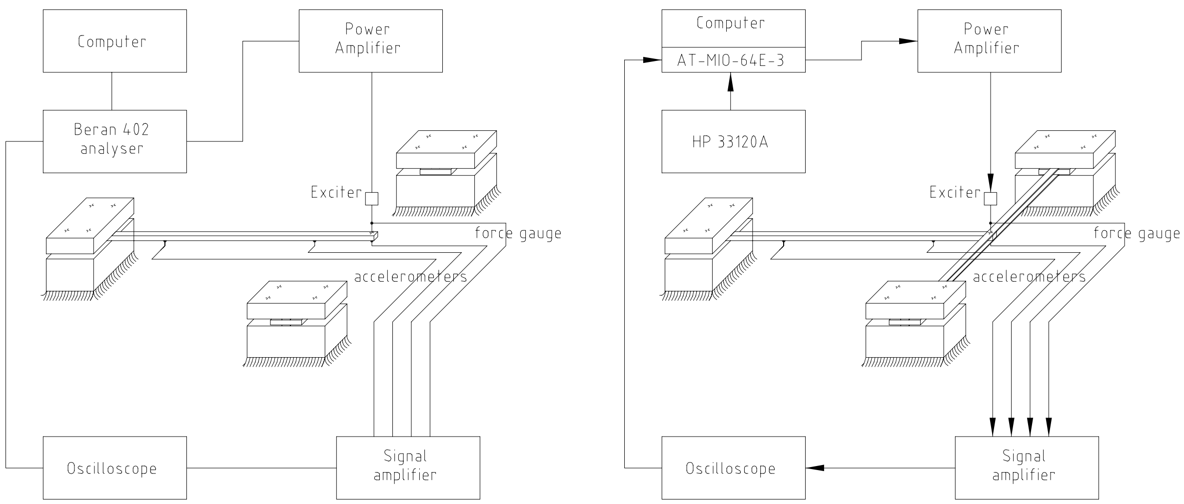
Figura 1: System configuration

section of 12mm by 8mm, which was clamped at one end and supported at the other end by two clamped-clamped beams, B1 and B2, of 380mm length with a cross section of 13mm by 1.5mm. The stiffness of the beam A is linear and the local nonlinear cubic stiffness is produced by two clamped-clamped beams, $B1$ and $B2$, due to the increase of the longitudinal tension under large amplitude of vibrations. Three accelerometers and one force-measuring transducer were attached along the beam. The first, second and third accelerometers were placed at 6mm, 82mm and 308mm respectively from the free end of the beam. The force transducer was located at 6mm from the free end of the beam. Both beams, $B1$ and $B2$, modelled as the nonlinear joint, were clamped to the beam A at the place of intersection as shown in Figure 1. They are bolted together on one side by the force transducer and on the other side by a block mass with a thread.

2.2. Experimental Setup

The experimental setup consisted of a shaker connected via a push-rod to a B&K 8200 force transducer which was used to measure the input force to the structure. All the resulting responses at selected points were measured using the ENDEVCO 2222C lightweight accelerometers which were attached to the structure using beeswax.

Two frequency response analysers were used to measure FRFs of the structures. The first one, the Beran 402 Frequency Response Analyser, was used to measure the linear FRFs. The second one, a "virtual analyser", was used to obtain FRFs of nonlinear structures when a special parameter is required to be controlled. The virtual analyser consisted of a Pentium 200MHz computer, a National Instruments card AT-MIO-64E-3, an HP 33120A function generator and the Intelligent Nonlinear Coupling Analysis software ($INCA^{++}$) (Ferreira 1998). The block diagram of the linear and nonlinear experimental setup using both analysers can be seen in Figures 2.a and 2.b, respectively.



(a) Linear experimental setup diagram

(b) Nonlinear experimental setup diagram

Figura 2: Block diagrams of the experimental setup Test Rig

The National Instruments card AT-MIO-64E-3 has 64 analogue input channels and two analogue outputs. The analogue output channel is used as an arbitrary waveform generator where the waveform generated can be updated on the fly when the card is driven by an external clock. The external clock used is via the HP 33120A function generator that has the capability of changing the clock rate of the square output signal without discontinuity, avoiding long settling time for the structure to achieve the steady-state response when changing to another frequency.

The $INCA^{++}$ software employs a correlation technique to calculate the amplitude and phase components of the fundamental and harmonics of excitation force and response signals. This technique requires a synchronisation between the excitation and the response, which is obtained by having both the system acquisition and the arbitrary wave form generator of the National Instruments card driven by one single external clock via the HP 33120A function generator.

Static and dynamic tests were done. The static test refers to obtaining the relationship between static loading and structure deformation while the dynamic test involves the measuring structure's FRFs. The displacement for the static test was obtained by a Solartron DF9150 LVDT transducer and the force was obtained by a Saxeway J7100-500N load cell with a Fylde FE-492-BBS bridge conditioner.

2.3. Nonlinear Force Control Algorithm

The nonlinear force algorithm can be derived by first representing the voltage applied in the shaker armature, $\boldsymbol{v}$, and the force in the transducer, $\boldsymbol{f}$, as Fourier series:

$$\boldsymbol{v} = \sum_{m=0}^{\infty} \boldsymbol{v}^m = \sum_{m=0}^{\infty} \boldsymbol{\mathcal{V}}^m e^{im\Psi} \tag{1}$$

$$\boldsymbol{f} = \sum_{m=0}^{\infty} \boldsymbol{f}^m = \sum_{m=0}^{\infty} \boldsymbol{\mathcal{F}}^m e^{im\Psi} \tag{2}$$

Considering n harmonic components in the force, the coefficients in equations (1,2) can be written in matricial form as

$$\boldsymbol{v} = \begin{Bmatrix} \boldsymbol{\mathcal{V}}^1 \\ \boldsymbol{\mathcal{V}}^2 \\ \vdots \\ \boldsymbol{\mathcal{V}}^n \end{Bmatrix} \qquad \boldsymbol{f} = \begin{Bmatrix} \boldsymbol{\mathcal{F}}^1 \\ \boldsymbol{\mathcal{F}}^2 \\ \vdots \\ \boldsymbol{\mathcal{F}}^n \end{Bmatrix} \tag{3}$$

The relation between the required force in the force transducer, $\boldsymbol{f}$, and the adjusted harmonic input voltage signal, $\boldsymbol{v}$, at frequency, ω, in the shaker armature can be expressed via an unknown functional relationship, $\boldsymbol{F}$, as:

$$\boldsymbol{f} = \boldsymbol{F}(\boldsymbol{v}, \omega) \tag{4}$$

If $(\boldsymbol{v}_d, \boldsymbol{f}_d)$ is the solution which satisfies equation (4), then:

$$\boldsymbol{f}_d - \boldsymbol{F}(\boldsymbol{v}_d, \omega) = 0 \tag{5}$$

Equation (5) can be solved by the Newton-Raphson method. This method gives a very efficient means of converging to a solution, given a sufficiently good initial guess. Therefore, it is desirable to find a good initial guess that can be used to find the solution and at the same time avoid the problem of estimating a wrong excitation force that will damage either the measurement equipment or the structure. Test experience showed that a good initial guess is the first harmonic solution. Thus, instead of finding a solution for a set of nonlinear system of equations, the problem is reduced to a one-dimensional nonlinear equation where the only concern is to first find a voltage $(\boldsymbol{\mathcal{V}})_1$ and ignore the higher harmonics. It is interesting to point out that for the first harmonic only the amplitude is controlled, whereas for the remaining harmonics, both the amplitude and phase must be controlled. After obtained the harmonic solution, the resulting solution is used as an initial guess for the Newton-Raphson method using now all the nonlinear equations (Ferreira 1998).

2.4. Measured and Updated FRF of Linear Test Rig Assembly

Due to different stiffness conditions which apply to the analytical and experimental configurations, an updated analytical model was used to represent the linear clamped beam as shown in Figure 3.

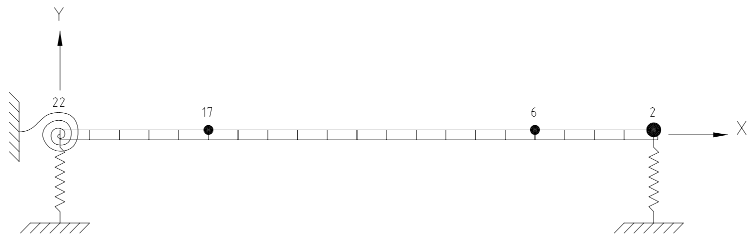
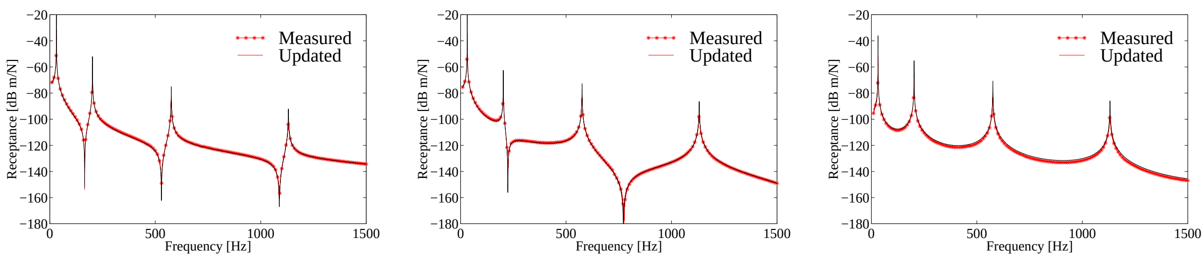


Figura 3: Analytical linear model Test Rig

The beam was modelled using Timoshenko beam elements with the updated values of elasticity modulus of $1.96E11\ N/m^2$ and density of $7900\ kg/m^3$. The updated translational and rotational springs used to model the clamped joint were $20E6\ N/m$ and $1E4\ N.m/rad$ respectively. The updated mass and moment of inertia used in nodes 6 and 17 were $1\,g$ and $3.8E-8\ Kg\,m^2$, respectively. The updated mass and moment of inertia used in node 2 were $37\,g$ and $9.6E-6\ Kgm^2$, respectively. The mass of the shaker was $46\ kg$ and the updated spring of the shaker used in node 2 was $250\ N/m$.

Three FRFs, H_{11}, H_{21} and H_{31}, were measured by $INCA^{++}$ software using the BERAN 402 frequency response analyser. The frequency range was from 20 Hz to 1500 Hz and the excitation frequency was increased by steps of 0.01 Hz. For all three FRFs, excitation was at node 2 and responses were measured at nodes 2, 6 and 17 respectively.

The measured and updated frequency response functions of the linear structure for the three positions along the beam are shown in Figures 4.a, 4.b, 4.c.



(a) Frequency Response H_{11} (b) Frequency Response H_{21} (c) Frequency Response H_{31}

Figura 4: Frequency Responses of linear structure

2.5. Experimental properties of the joints

The properties of the nonlinear joints were obtained by first performing a static test. After the static test was done, a curve of the type $k1 * x + k2 * x^3$ was fitted and the parameters $k1 = 6500\ N/m$ and $k2 = 11800E5\ N/m$ were obtained. The same beam was modelled in ANSYS and the relationship between force and deformation for all these cases is shown in Figure 5.

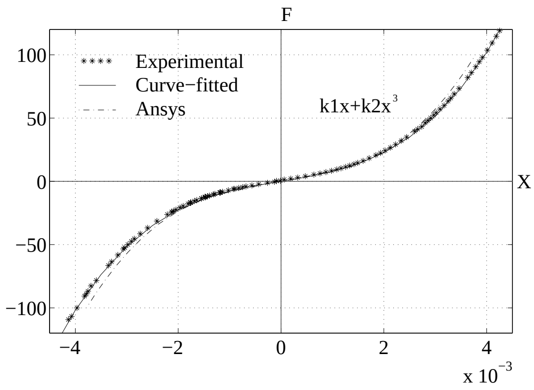


Figura 5: Relationship between loading and deformation

2.6. Nonlinear Test Rig Assembly Model

The analytical model used to represent the nonlinear dynamic test rig is shown in Figure 6.

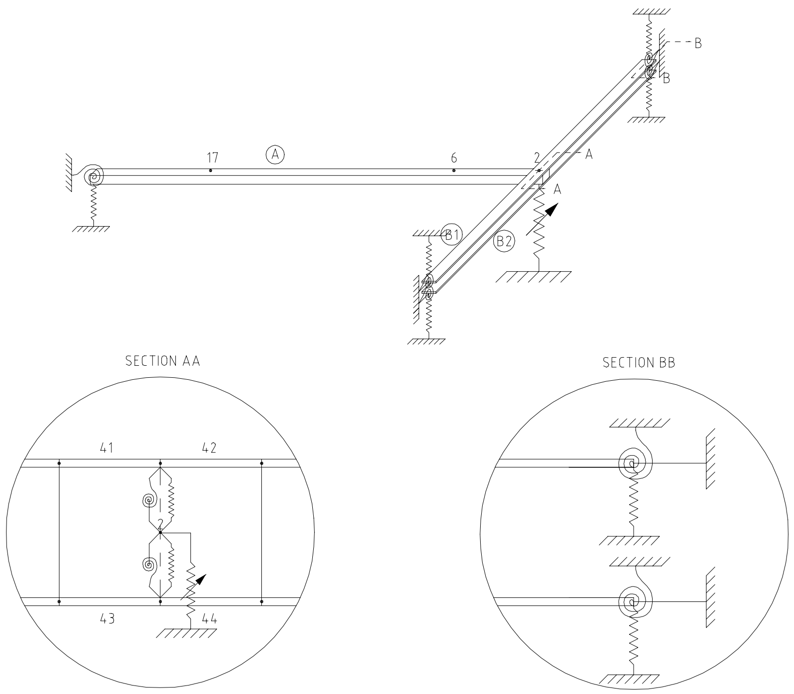
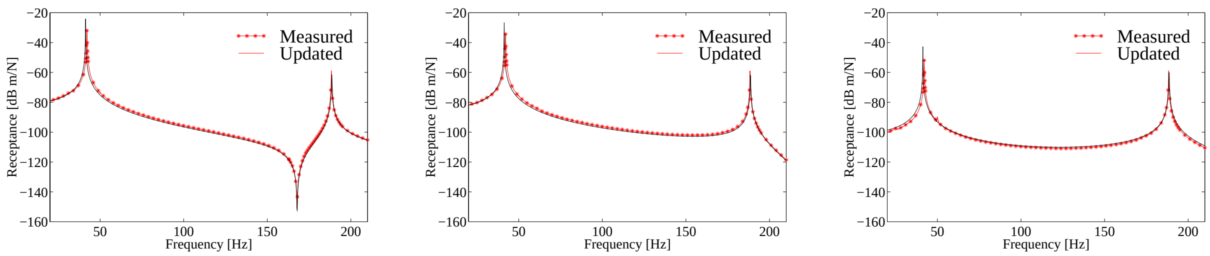


Figura 6: Analytical model Test Rig

This assembly model consists of two substructures coupled to each other at node 2. The first substructure composed of the beam A is modelled as a linear clamped-free beam, discretised as 21 2D Timoshenko beam elements. The second substructure, composed of the beams $B1$ and $B2$, is modelled as two parallel linear clamped-clamped beams each one discretised with 20 2D Timoshenko beam elements. The nonlinear behaviour of the substructure is modelled as a concentrated nonlinear massless spring joint. The assembly structure was updated by changing only parameters from the second substructure, once the first substructure was already updated in section 2.4.. This is equivalent to updating the linear behaviour of the nonlinear beams. The linear response of the assembled structure was measured assuming that for low amplitudes of vibration, without controlling the force, the FRF measured is the closest linear representation of the linear dynamic behaviour.

The beams were modelled with the updated values of elasticity modulus of $2.3E11$ N/m^2 and density of 7800 kg/m^3. The stiffnesses of the translational spring in the y direction and the rotational spring in relation to the x axis used to model the clamped condition in both ends of the beams were $1E6$ N/m and 500 Nm/rad, respectively. In order to represent the boundary conditions imposed by clamping the nonlinear beams to both sides of the linear beam, the stiffnesses of the beam elements 41,42,43 and 44 were increased by increasing the section area. The updated value of the area is $4.3E-5$ m^2. The bolt that connects the linear beam with the nonlinear beam was modelled using translational and rotational springs. The updated value for the translational spring stiffness was $2.2E6$ N/m and the rotational spring, 9 Nm/rad. The updating was repeated until the following two conditions were met. First, the anti-resonance and resonance of the second mode were in good agreement with the experimental FRF. Second, the reciprocal of an element of the flexibility matrix (which has been measured) must be 6500 N/m (2.5.). This element is related to node 2 and its excitation and response in the y-direction. No attempt was made to update the model to match the natural frequency of the first mode because of its strong nonlinearity.

The result of the updated linear assembly structure for nodes 2, 6 and 17 can be seen in figures (7.a, 7.b, 7.c).

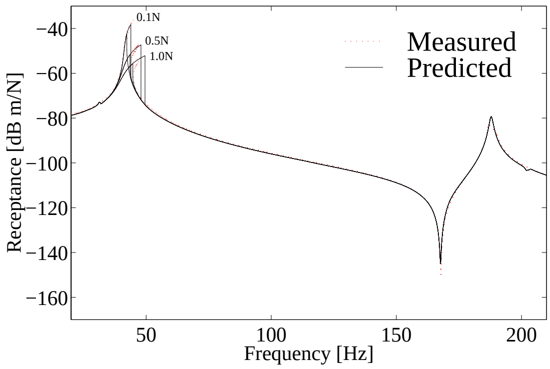


(a) Frequency Response H_{11} (b) Frequency Response H_{21} (c) Frequency Response H_{31}

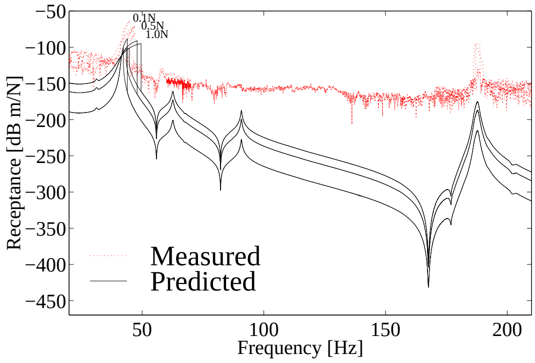
Figura 7: Frequency Responses of the coupled linear structure

2.7. Measured and Predicted Coupling FRFs using First Assembly Model Test Rig

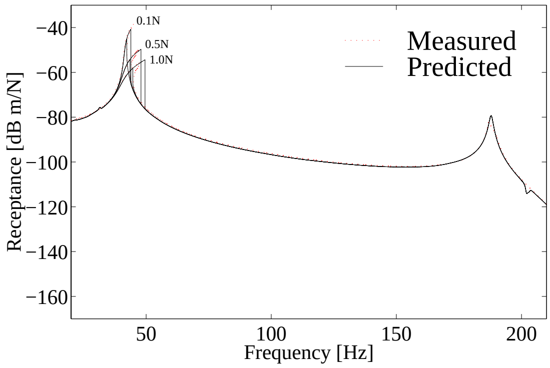
Using the joint parameters obtained in section 2.5. in the analytical model presented in section 2.6., the frequency response functions $\{H_{11}^{11}\}^3$, $\{H_{11}^{31}\}^3$, $\{H_{21}^{11}\}^3$, $\{H_{21}^{31}\}^3$, $\{H_{31}^{11}\}^3$ and $\{H_{31}^{31}\}^3$ for forces of 0.1N, 0.5N and 1N were predicted by the $INCA^{++}$ software using the $MUHANORCA$ method. Figure 8.a shows the FRF $\{H_{11}^{11}\}^3$ data measured from point 2 within a certain frequency range and the corresponding predicted FRF data. Figure 8.b shows the FRF $\{H_{11}^{31}\}^3$ measured from point 2 and the corresponding FRF predicted data. Figure 8.c shows the FRF $\{H_{21}^{11}\}^3$ measured from point 6 and the corresponding FRF predicted data. Figure 8.d shows the FRF $\{H_{21}^{31}\}^3$ measured from point 6 and the corresponding FRF predicted data. Figure 8.e shows the FRF $\{H_{31}^{11}\}^3$ measured from point 17 and the corresponding FRF predicted data. Figure 8.f shows the FRF $\{H_{31}^{31}\}^3$ measured from point 17 and the corresponding FRF predicted data.



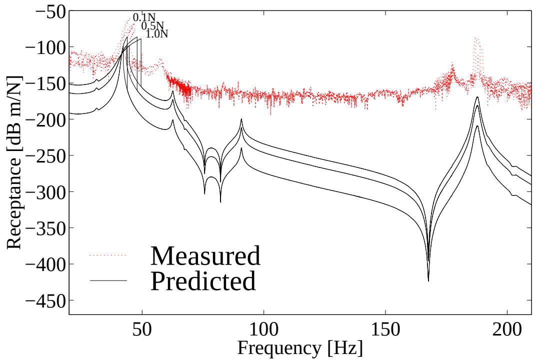
(a) Frequency Response $\{H_{11}^{11}\}^3$



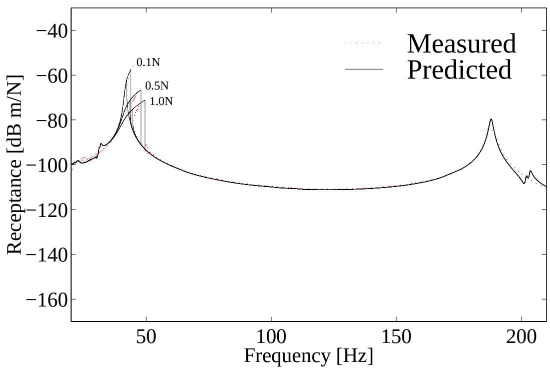
(b) Frequency Response $\{H_{11}^{31}\}^3$



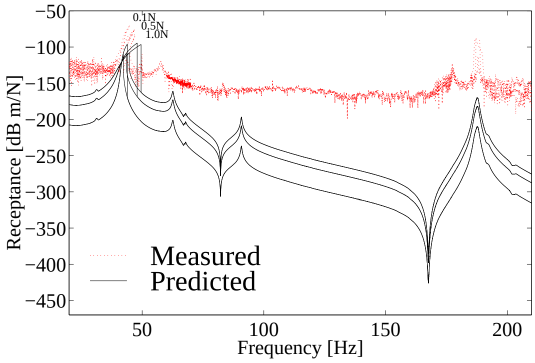
(c) Frequency Response $\{H_{21}^{11}\}^3$



(d) Frequency Response $\{H_{21}^{31}\}^3$



(e) Frequency Response $\{H_{31}^{11}\}^3$



(f) Frequency Response $\{H_{31}^{31}\}^3$

Figura 8: Frequency Responses at nodes 2,6,17

2.8. Discussion of Results

The analytical model presented in section 2.6. succeeded in predicting the dynamic behaviour of the coupled structure in the frequency range measured. The predicted nonlinear behaviour was calculated by using the measured FRFs and the analytical FRFs obtained from the updated models. The analytical FRFs were used in substitution of the FRFs that were not measured but were required in the coupling procedure. Various updated models were obtained for the cantilever beam but the result predictions were not good because although it has a good representation of the dynamic behaviour for the frequency range measured, the natural frequencies were close but not close enough. Small shifts in the resonance and anti-resonance imply in errors in the prediction. The predicted assembly first-order FRFs obtained for the frequency range including the first and second mode agree well with those measured in the assembled structure. These good results were obtained since the model updated for the cantilever beam was able to have resonance and anti-resonance very close to the measured physical model. On the other hand, for the higher-order FRFs the result present a shifting problem, which makes it possible to be correlated with a systematic error occurred in the measurements. From the predicted higher order FRFs is possible to observe that the level of the harmonic response is very small which implies that for the cubic stiffness nonlinearity the influence of these higher harmonics is very small. For cubic stiffness nonlinearity, mathematics shows that there are three response solutions for some frequencies. The conventional idea is that two response solutions are stable and one that is unstable. The two stable solutions are usually obtained by measuring the response from frequency sweeps up and down. According to the conventional idea, the unstable one is impossible to measure. No other known recorded results show the measurements of the unstable solution. Therefore it was a surprise to be able to "measure" all three cases.

A reasonable explanation is as follows. The assembly structure can be considered as a sum of two substructures: the nonlinear structure itself and the shaker-push-rod system. Although the former has the unstable response, when put together, the assembly is stable. Therefore using the transducer force between both substructures and the accelerometer in the nonlinear substructure, it is possible to measure a stable response of the assembly which corresponds to the unstable response of the nonlinear substructure.

3. CONCLUDING REMARKS

Application of the nonlinear FRF-based coupling technique, $MUHANORCA$, to a practical structure has been examined in this paper.

For the first-order FRFs, the overall shape of the predicted FRFs match the measured counterparts very well. However, for the higher-order FRFs, the difference between the predicted and measured FRFs of the assembly structure is much more significant. It is believed that the true FRF cannot be measured as the high level of noise overwrites the dynamic behaviour of the structure. Therefore for low amplitudes of vibration, the measurements are constrained to noise. For large amplitudes of vibration, the measurements follow the overall trend quite well, as the resonances are reasonably well captured.

A very interesting result was the possibility of measuring FRFs in the unstable area, although this was thought to be impossible. A reasonable explanation of this fact is that although the nonlinear structure itself has an unstable response, the structure composed of the shaker plus the nonlinear structure is now stable.

The $MUHANORCA$ has been successfully verified for the stationary structure.

4. ACKNOWLEDGEMENTS

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